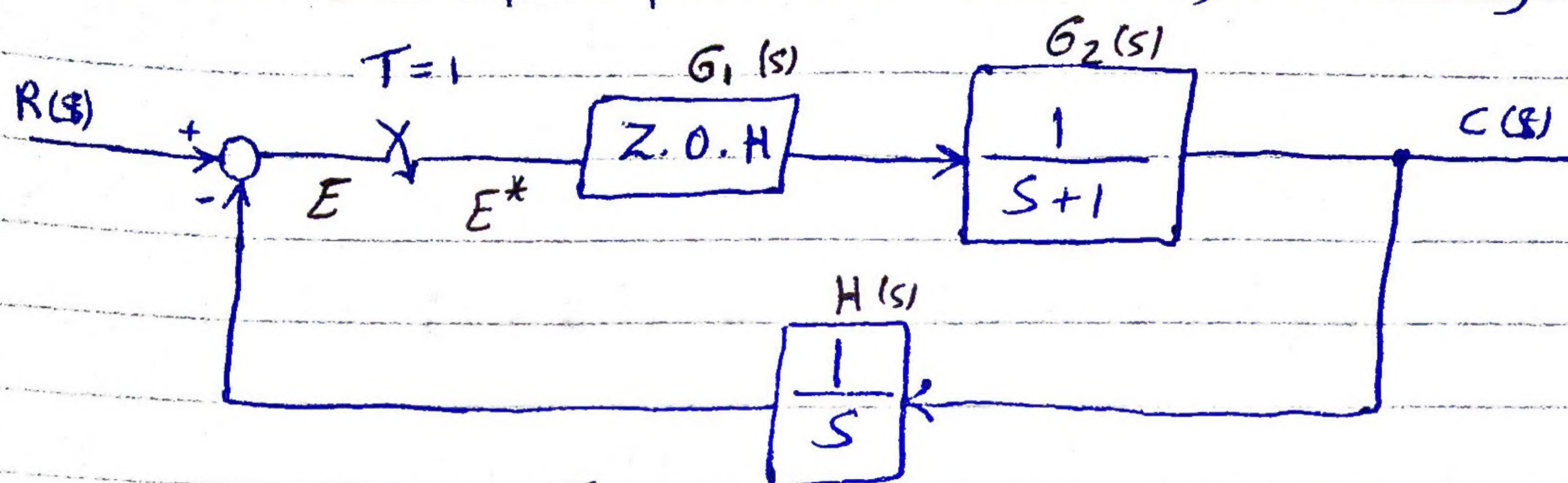


1 Find the unit-step response and discuss system stability



$$G_{Z.O.H}(s) = \frac{1 - e^{-sT}}{s}$$

$$C(s) = E^* G_1 G_2(s) \xrightarrow{\text{Sampling}} C^*(s) = E^* \overline{G_1 G_2}^*(s) \rightarrow (1)$$

$$E(s) = R(s) - C(s) H(s) \\ = R(s) - E^* G_1 G_2 H(s) \xrightarrow{\text{Sampling}} E^*(s) = R^*(s) - E^*(s) \overline{G_1 G_2 H}^*(s) \rightarrow (2)$$

$$\text{From (2)} \rightarrow E^*(s) (1 + \overline{G_1 G_2 H}^*(s)) = R^*(s) \rightarrow \text{in (1)}$$

$$C^*(s) = \frac{R^*(s) \overline{G_1 G_2}^*(s)}{1 + \overline{G_1 G_2 H}^*(s)}$$

$$C(z) = \frac{R(z) \overline{G_1 G_2}(z)}{1 + \overline{G_1 G_2 H}(z)} \rightarrow (3)$$

$$\overline{G_1 G_2}(z) = \mathcal{Z} \left[\frac{1 - e^{-sT}}{s} \cdot \frac{1}{s+1} \right] = (1 - z^{-1}) \mathcal{Z} \left[\frac{1}{s(s+1)} \right]$$

$$= (1 - z^{-1}) \mathcal{Z} \left[\frac{1}{s} + \frac{-1}{s+1} \right]$$

$$= (1 - z^{-1}) \mathcal{Z} [u(t) - e^{-t}]$$

$$= (1 - z^{-1}) \left[\frac{z}{z-1} - \frac{z}{z - e^{-1}} \right] \rightarrow (4)$$

$$= 1 - \frac{z-1}{z - e^{-1}} = \frac{z - e^{-1} - (z-1)}{z - e^{-1}}$$

$$e^{-1} = 0.368$$

$$= \frac{0.632}{z - 0.368} \rightarrow (4)$$

$$\begin{aligned}
 \overline{G_1 G_2 H}(z) &= Z \left[\frac{1 - e^{-sT}}{s} \cdot \frac{1}{s+1} \cdot \frac{1}{s} \right] = (1 - z^{-1}) Z \left[\frac{1}{s^2(s+1)} \right] \\
 &= (1 - z^{-1}) Z \left[\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1} \right] \\
 &= (1 - z^{-1}) Z [t - u(t) + e^{-t}] \\
 &= (1 - z^{-1}) \left[\frac{z}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-e^{-1}} \right] \\
 &= \frac{1}{z-1} - 1 + \frac{z-1}{z-0.368} \\
 &= \frac{0.368z + 0.264}{(z-1)(z-0.368)} \rightarrow \textcircled{5}
 \end{aligned}$$

From $\textcircled{4}, \textcircled{5}$ in $\boxed{3}$

$$\begin{aligned}
 C(z) &= R(z) \frac{\frac{0.632}{z-0.368}}{1 + \frac{0.368z + 0.264}{(z-1)(z-0.368)}} \\
 &= R(z) \frac{0.632(z-1)}{(z-1)(z-0.368) + 0.368z + 0.264} \\
 &= R(z) \frac{0.632(z-1)}{z^2 - 1.368 + 0.368 + 0.368z + 0.264} \\
 &= R(z) \frac{0.632(z-1)}{z^2 - z + 0.632}
 \end{aligned}$$

Unit step response

$$C(z) = \frac{z}{z-1} \frac{0.632(z-1)}{z^2 - z + 0.632} = \frac{0.632z}{z^2 - z + 0.632}$$

~~$$2 \cos(60^\circ) = 1$$~~

~~$$\cos(60^\circ) = 0.5$$~~

~~$$\omega = 60^\circ = \frac{\pi}{3} \text{ rad}$$~~

~~$$\sin(60^\circ) = 0.866$$~~

$$C(z) = \frac{0.632 z}{z^2 - z + 0.632}$$

$$\sin(\omega t) \rightarrow \frac{\sin(\omega) z}{z^2 - 2 \cos(\omega) z + 1}$$

$$\begin{aligned} a^t \sin(\omega t) &\rightarrow \frac{\sin(\omega) z / a}{\frac{z^2}{a^2} - 2 \cos(\omega) \frac{z}{a} + 1} \\ &= \frac{a \sin(\omega) z}{z^2 - 2a \cos(\omega) z + a^2} \end{aligned}$$

$$a^2 = 0.632 \rightarrow a = 0.795$$

$$-2a \cos(\omega) = -1$$

$$\cos(\omega) = \frac{1}{2a} = 0.628$$

$$\omega = 51.0278$$

$$\sin(\omega) = 0.777$$

$$\begin{aligned} C(z) &= (0.795)^t \sin(51.027^\circ t) \left[\frac{0.632}{0.795 * 0.777} \right] \\ &= (1.023) (0.795)^t \sin(51.027^\circ t) \end{aligned}$$

Study stability

$$S.S.e = R(t) - C(\infty)$$

$$= 1 - \lim_{z \rightarrow 1} C(z) (z-1)$$

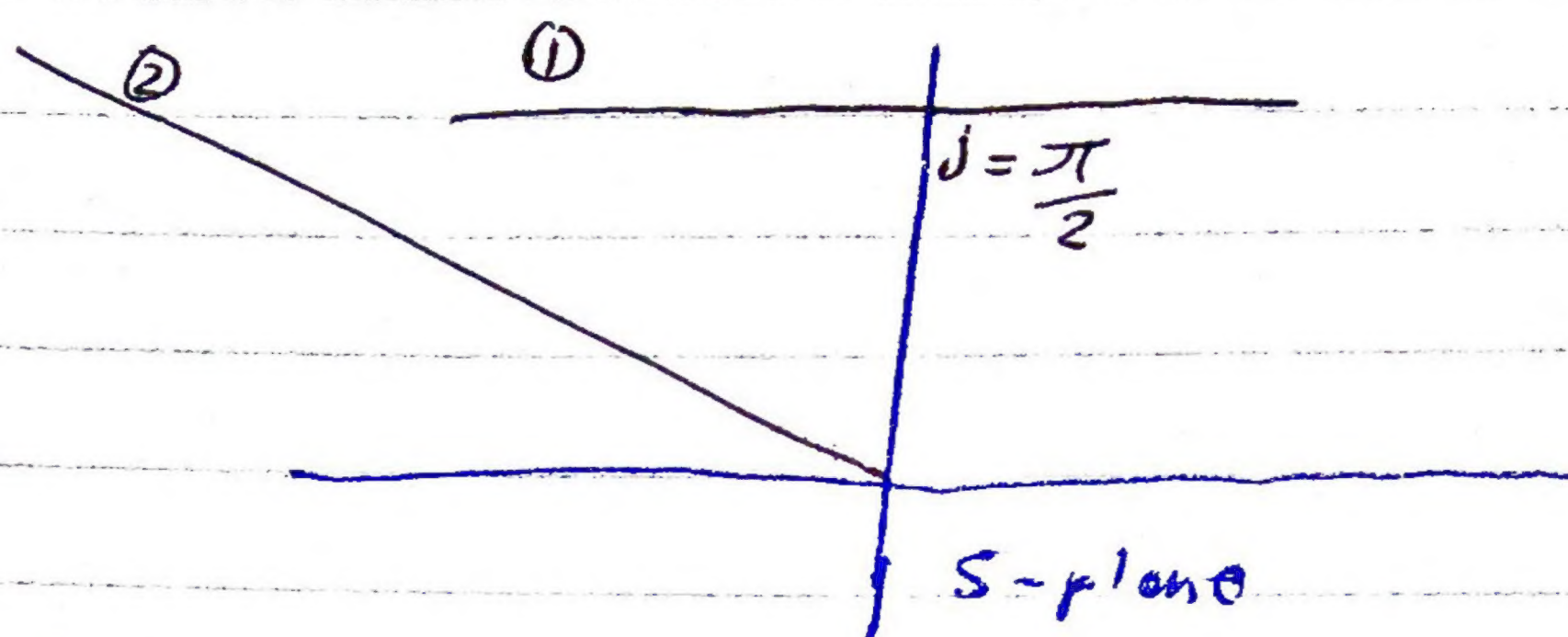
$$= 1 - \lim_{z \rightarrow 1} (z-1) \frac{0.632 z}{z^2 - z + 0.632}$$

$$S.S.e = 1$$

There is an error equal 100 % of the desired output

So system is not stable

2 map to Z-plane



For Line ①


- σ changes from $-\infty$ to ∞
- So $r = e^{\sigma}$ changes from 0 to ∞
- ω has a fixed value equal $\frac{\pi}{2} = 90^\circ$
- So $\theta = \omega = \frac{\pi}{2}$

We can represent that with line with angle $\frac{\pi}{2}$
Start from 0 to ∞ in radius



For Line ②

- Line lies in the left side
- So $r \leq 0$
- σ changes from 0 to $-\infty$
- So r changes from ∞ to 0
- ω changes from 0 to ∞
- So θ changes from 0 to ∞

We can represent that with  that start with $r = \infty$ with $\theta = 0$

and end with $r = 0$
angle becomes more positive

So curve move
to the positive direction
angle

